

ON THE q -GENOCCHI NUMBERS AND POLYNOMIALS WITH WEIGHT ZERO AND THEIR APPLICATIONS

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ABSTRACT. In this paper, the authors deal with the q -Genocchi numbers and polynomials with weight zero. They discover some interesting relations via the p -adic q -integral on \mathbb{Z}_p and familiar basis Bernstein polynomials. Finally, the authors show that the p -adic log gamma functions are associated with the q -Genocchi numbers and polynomials with weight zero.

1. PRELIMINARIES

Let p be an odd prime number. Denote the ring of the p -adic rational integers by \mathbb{Z}_p , the field of rational numbers by \mathbb{Q} , the field of the p -adic rational numbers by \mathbb{Q}_p , and the completion of algebraic closure of \mathbb{Q}_p by \mathbb{C}_p , respectively. Let \mathbb{N} be the set of positive integers and $\mathbb{N}^* = \mathbb{N} \cup \{0\}$ the set of all non-negative integers. The p -adic absolute value is defined by

$$|p|_p = \frac{1}{p}. \quad (1.1)$$

Assume $|q - 1|_p < 1$ is an indeterminate number in the sense that either $q \in \mathbb{C}$ or $q \in \mathbb{C}_p$. A q -analogue of x may be defined by

$$[x]_q = \frac{1 - q^x}{1 - q} \quad (1.2)$$

satisfying $\lim_{q \rightarrow 1} [x]_q = x$.

A function f is said to be uniformly differentiable at a point $a \in \mathbb{Z}_p$ if the divided difference

$$F_f(x, y) = \frac{f(x) - f(y)}{x - y}$$

converges to $f'(a)$ as $(x, y) \rightarrow (a, a)$. The class of all the uniformly differentiable functions is denoted by $UD(\mathbb{Z}_p)$.

For $f \in UD(\mathbb{Z}_p)$, the p -adic q -analogue of Riemann sum for f was defined by

$$\frac{1}{[p^n]_q} \sum_{0 \leq \xi < p^n} f(\xi) q^\xi = \sum_{0 \leq \xi < p^n} f(\xi) \mu_q(\xi + p^n \mathbb{Z}_p) \quad (1.3)$$

in [6, 8], where $n \in \mathbb{N}$. The integral of f on \mathbb{Z}_p is defined as the limit of (1.3) as n tends to ∞ , if it exists, and represented by

$$I_q(f) = \int_{\mathbb{Z}_p} f(\xi) d\mu_q(\xi). \quad (1.4)$$

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The bosonic integral and the fermionic p -adic integral on \mathbb{Z}_p are defined respectively by

$$I_1(f) = \lim_{q \rightarrow 1} I_q(f) \quad (1.5)$$

and

$$I_{-q}(f) = \lim_{q \rightarrow -q} I_q(f). \quad (1.6)$$

For a prime p and a positive integer d with $(p, d) = 1$, set

$$X = X_d = \varprojlim \mathbb{Z}/dp^n\mathbb{Z}, \quad X_1 = \mathbb{Z}_p, \\ X^* = \bigcup_{\substack{(a,p)=1 \\ 0 < a < dp}} a + dp\mathbb{Z}_p,$$

and

$$a + dp^n\mathbb{Z}_p = \{x \in X \mid x \equiv a \pmod{dp^n}\},$$

where $a \in \mathbb{Z}$ satisfies $0 \leq a < dp^n$ and $n \in \mathbb{N}$.

2. MAIN RESULTS

In [1, 2], Aracı, Açıkgöz, and Seo considered the q -Genocchi polynomials with weight α in the form

$$\frac{\tilde{G}_{n+1,q}^{(\alpha)}(x)}{n+1} = \int_{\mathbb{Z}_p} [x + \xi]_{q^\alpha}^n d\mu_{-q}(\xi), \quad (2.1)$$

where $\tilde{G}_{n+1,q}^{(\alpha)} = \tilde{G}_{n+1,q}^{(\alpha)}(0)$ is called the q -Genocchi numbers with weight α . Taking $\alpha = 0$ in (2.1), we easily see that

$$\frac{\tilde{G}_{n+1,q}}{n+1} \triangleq \frac{\tilde{G}_{n+1,q}^{(0)}}{n+1} = \int_{\mathbb{Z}_p} \xi^n d\mu_{-q}(\xi), \quad (2.2)$$

where $\tilde{G}_{n,q}$ are called the q -Genocchi numbers and polynomials with weight 0. From (2.2), it is simple to see

$$\sum_{n=0}^{\infty} \tilde{G}_{n,q} \frac{t^n}{n!} = t \int_{\mathbb{Z}_p} e^{\xi t} d\mu_{-q}(\xi). \quad (2.3)$$

By (1.6), we have

$$q^n I_{-q}(f_n) + (-1)^{n-1} I_{-q}(f) = [2]_q \sum_{0 \leq \ell < n} q^\ell (-1)^{n-1-\ell} f(\ell), \quad (2.4)$$

where $f_n(x) = f(x+n)$ and $n \in \mathbb{N}$. See [5, 7, 9]. Taking $n = 1$ in (2.4) leads to the well-known equality

$$q I_{-q}(f_1) + I_{-q}(f) = [2]_q f(0), \quad (2.5)$$

When setting $f(x) = e^{xt}$ in (2.5), we find

$$\sum_{n=0}^{\infty} \tilde{G}_{n,q} \frac{t^n}{n!} = \frac{[2]_q t}{qe^t + 1}. \quad (2.6)$$

By (2.6), we obtain the q -Genocchi polynomials with weight 0 as follows

$$\sum_{n=0}^{\infty} \tilde{G}_{n,q}(x) \frac{t^n}{n!} = \frac{[2]_q t}{qe^t + 1} e^{xt}. \quad (2.7)$$

By (2.7), we see that

$$\sum_{n \geq 0} \tilde{G}_{n,q}(x) \frac{t^n}{n!} = t \frac{1 - (-q^{-1})}{e^t - (-q^{-1})} e^{xt} = t \sum_{n \geq 0} H_n(-q^{-1}, x) \frac{t^n}{n!}.$$

By equating coefficients of t^n on both sides of the above equality, we derive the following theorem.

Theorem 1. *For $n \in \mathbb{N}$, we have*

$$\frac{\tilde{G}_{n+1,q}(x)}{n+1} = H_n(-q^{-1}, x),$$

where $H_n(-q^{-1}, x)$ are the n -th Frobenius-Euler polynomials.

By (2.5), we discover that

$$\begin{aligned} [2]_q \sum_{n=0}^{\infty} x^n \frac{t^n}{n!} &= q \int_{\mathbb{Z}_p} e^{(x+\xi+1)t} d\mu_{-q}(\xi) + \int_{\mathbb{Z}_p} e^{(x+\xi)t} d\mu_{-q}(\xi) \\ &= \sum_{n=0}^{\infty} \left[q \int_{\mathbb{Z}_p} (x+\xi+1)^n d\mu_{-q}(\xi) + \int_{\mathbb{Z}_p} (x+\xi)^n d\mu_{-q}(\xi) \right] \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} [qH_n(-q^{-1}, x+1) + H_n(-q^{-1}, x)] \frac{t^n}{n!}. \end{aligned}$$

Equating coefficients of $\frac{t^n}{n!}$ on both sides above equation, we deduce the following theorem.

Theorem 2. *For $n \in \mathbb{N}$, the identity*

$$qH_n(-q^{-1}, x+1) + H_n(-q^{-1}, x) = [2]_q x^n \quad (2.8)$$

is valid.

In particular, when letting $q = 1$, the identity (2.8) becomes

$$G_n(x+1) + G_n(x) = 2nx^{n-1}, \quad (2.9)$$

where $G_n(x)$ are called the Genocchi polynomials.

If we substitute $x = 0$ into (2.8), then Theorem 2 can be rewritten as Theorem 3 below.

Theorem 3. *The identity*

$$q\tilde{G}_{n,q}(1) + \tilde{G}_{n,q} = \begin{cases} [2]_q, & n = 1 \\ 0, & n \neq 1 \end{cases} \quad (2.10)$$

is true, where $\tilde{G}_{n,q}$ are called the Genocchi numbers and polynomials with weight 0.

When we substitute x by $1-x$ and q by q^{-1} in (2.7), it follows that

$$\begin{aligned} \sum_{n=0}^{\infty} \tilde{G}_{n,q^{-1}}(1-x) \frac{t^n}{n!} &= t \frac{1+q^{-1}}{q^{-1}e^t+1} e^{(1-x)t} = \frac{1+q}{e^t+q} e^t e^{xt} \\ &= -\frac{[2]_q(-t)}{qe^{-t}+1} e^{(-t)x} = \sum_{n=0}^{\infty} (-1)^{n+1} \tilde{G}_{n,q}(x) \frac{t^n}{n!}. \end{aligned}$$

From this, we procure symmetric properties of this type polynomials.

Theorem 4. *The following identity holds*

$$\tilde{G}_{n,q^{-1}}(1-x) = (-1)^{n+1} \tilde{G}_{n,q}(x). \quad (2.11)$$

By using (2.1) for $\alpha = 0$ and the binomial theorem, we readily obtain that

$$\begin{aligned} \frac{\tilde{G}_{n+1,q}(x)}{n+1} &= \int_{\mathbb{Z}_p} (x+\xi)^n d\mu_{-q}(\xi) \\ &= \sum_{k=0}^n \binom{n}{k} \left[\int_{\mathbb{Z}_p} \xi^k d\mu_{-q}(\xi) \right] x^{n-k} \\ &= \sum_{k=0}^n \binom{n}{k} \frac{\tilde{G}_{k+1,q}}{k+1} x^{n-k}. \end{aligned}$$

Further using

$$\frac{n+1}{k+1} \binom{n}{k} = \binom{n+1}{k+1},$$

we obtain

$$\tilde{G}_{n+1,q}(x) = \sum_{k=0}^n \binom{n+1}{k+1} \tilde{G}_{k+1,q} x^{n-k} = \sum_{k=1}^{n+1} \binom{n+1}{k} \tilde{G}_{k,q} x^{n+1-k}.$$

Thus, we get the following conclusion.

Theorem 5. *The identity*

$$\tilde{G}_{n,q}(x) = \sum_{k=0}^n \binom{n}{k} \tilde{G}_{k,q} x^{n-k} \quad (2.12)$$

is true, where the usual convention of replacing $(\tilde{G}_q)^n$ by $\tilde{G}_{n,q}$ is used.

Combining (2.10) with (2.12) leads to the following proposition.

Proposition 1. *The identity*

$$\tilde{G}_{0,q} = 0 \quad \text{and} \quad q(\tilde{G}_q + 1)^n + \tilde{G}_{n,q} = \begin{cases} [2]_q, & n = 1 \\ 0, & n \neq 1 \end{cases} \quad (2.13)$$

is true, where the usual convention of replacing $(\tilde{G}_q)^n$ by $\tilde{G}_{n,q}$ is used.

From (2.12), it follows that

$$\begin{aligned} q^2 \tilde{G}_{n+1,q}(2) &= q^2 (\tilde{G}_q + 1 + 1)^{n+1} \\ &= q^2 \sum_{k=0}^{n+1} \binom{n+1}{k} (\tilde{G}_q + 1)^k \\ &= (n+1)q^2 (\tilde{G}_q + 1)^1 + q \sum_{k=2}^{n+1} \binom{n+1}{k} q (\tilde{G}_q + 1)^k \\ &= (n+1)q([2]_q - \tilde{G}_{1,q}) - q \sum_{k=2}^{n+1} \binom{n+1}{k} \tilde{G}_{k,q} \\ &= (n+1)q[2]_q - \left[q \sum_{k=2}^{n+1} \binom{n+1}{k} \tilde{G}_{k,q} + (n+1)q\tilde{G}_{1,q} \right] \end{aligned}$$

$$\begin{aligned}
&= (n+1)q[2]_q - q \sum_{k=0}^{n+1} \binom{n+1}{k} \tilde{G}_{k,q} \\
&= (n+1)q[2]_q - q(\tilde{G}_q + 1)^{n+1} \\
&= (n+1)q[2]_q + \tilde{G}_{n+1,q}
\end{aligned}$$

for $n > 1$. Therefore, we deduce the following proposition.

Proposition 2. *For $n > 1$,*

$$\tilde{G}_{n+1,q}(2) = \frac{(n+1)}{q}[2]_q + \frac{1}{q^2}\tilde{G}_{n+1,q}. \quad (2.14)$$

By virtue of (1.6), (2.11), and (2.14), we find

$$\begin{aligned}
(n+1) \int_{\mathbb{Z}_p} (1-\xi)^n d\mu_{-q}(\xi) &= (n+1)(-1)^n \int_{\mathbb{Z}_p} (\xi-1)^n d\mu_{-q}(\xi) \\
&= (-1)^n \tilde{G}_{n+1,q}(-1) = \tilde{G}_{n+1,q^{-1}}(2) = (n+1)[2]_q + q^2 \tilde{G}_{n+1,q^{-1}}.
\end{aligned}$$

As a result, we may concluded Theorem 6 below.

Theorem 6. *The identity*

$$\int_{\mathbb{Z}_p} (1-\xi)^n d\mu_{-q}(\xi) = [2]_q + q^2 \frac{\tilde{G}_{n+1,q^{-1}}}{n+1} \quad (2.15)$$

is valid.

Let $UD(\mathbb{Z}_p)$ be the space of continuous functions on \mathbb{Z}_p . For $f \in UD(\mathbb{Z}_p)$, the p -adic analogue of Bernstein operator for f is defined by

$$B_n(f, x) = \sum_{k=0}^n f\left(\frac{k}{n}\right) B_{k,n}(x) = \sum_{k=0}^n f\left(\frac{k}{n}\right) \binom{n}{k} x^k (1-x)^{n-k},$$

where $n, k \in \mathbb{N}^*$ and the p -adic Bernstein polynomials of degree n is defined by

$$B_{k,n}(x) = \binom{n}{k} x^k (1-x)^{n-k}, \quad x \in \mathbb{Z}_p. \quad (2.16)$$

See [3, 10, 11, 12]. Via the p -adic q -integral on \mathbb{Z}_p and Bernstein polynomials in (2.16), we can obtain that

$$\begin{aligned}
I_1 &= \int_{\mathbb{Z}_p} B_{k,n}(\xi) d\mu_{-q}(\xi) \\
&= \binom{n}{k} \int_{\mathbb{Z}_p} \xi^k (1-\xi)^{n-k} d\mu_{-q}(\xi) \\
&= \binom{n}{k} \sum_{\ell=0}^{n-k} \binom{n-k}{\ell} (-1)^\ell \left[\int_{\mathbb{Z}_p} \xi^{\ell+k} d\mu_{-q}(\xi) \right] \\
&= \binom{n}{k} \sum_{\ell=0}^{n-k} \binom{n-k}{\ell} (-1)^\ell \frac{\tilde{G}_{\ell+k+1,q}}{\ell+k+1}.
\end{aligned}$$

On the other hand, by symmetric properties of Bernstein polynomials, we have

$$I_2 = \int_{\mathbb{Z}_p} B_{n-k,n}(1-\xi) d\mu_{-q}(\xi)$$

$$\begin{aligned}
&= \binom{n}{k} \sum_{s=0}^k \binom{k}{s} (-1)^{k+s} \int_{\mathbb{Z}_p} (1-\xi)^{n+s} d\mu_{-q}(x) \\
&= \binom{n}{k} \sum_{s=0}^k \binom{k}{s} (-1)^{k+s} \left([2]_q + q^2 \frac{\tilde{G}_{n+s+1, q^{-1}}}{n+s+1} \right) \\
&= \begin{cases} [2]_q + q^2 \frac{\tilde{G}_{n+1, q^{-1}}}{n+1}, & k=0 \\ \binom{n}{k} \sum_{s=0}^k \binom{k}{s} (-1)^{k+s} \left([2]_q + q^2 \frac{\tilde{G}_{n+s+1, q^{-1}}}{n+s+1} \right), & k \neq 0. \end{cases}
\end{aligned}$$

Equating I_1 and I_2 yields Theorem 7 below.

Theorem 7. *The following identity holds:*

$$\sum_{\ell=0}^{n-k} \binom{n-k}{\ell} (-1)^\ell \frac{\tilde{G}_{\ell+k+1, q}}{\ell+k+1} = \begin{cases} [2]_q + q^2 \frac{\tilde{G}_{n+1, q^{-1}}}{n+1}, & k=0; \\ \sum_{s=0}^k \binom{k}{s} (-1)^{k+s} \left([2]_q + q^2 \frac{\tilde{G}_{n+s+1, q^{-1}}}{n+s+1} \right), & k \neq 0. \end{cases}$$

The p -adic q -integral on \mathbb{Z}_p of the product of several Bernstein polynomials can be calculated as

$$\begin{aligned}
I_3 &= \int_{\mathbb{Z}_p} \prod_{s=1}^m B_{k, n_s}(\xi) d\mu_{-q}(\xi) \\
&= \prod_{s=1}^m \binom{n_s}{k} \int_{\mathbb{Z}_p} \xi^{mk} (1-\xi)^{n_1+\dots+n_m-mk} d\mu_{-q}(\xi) \\
&= \prod_{s=1}^m \binom{n_s}{k} \sum_{\ell=0}^{n_1+\dots+n_m-mk} \binom{n_1+\dots+n_m-mk}{\ell} (-1)^\ell \left[\int_{\mathbb{Z}_p} \xi^{\ell+mk} d\mu_{-q}(\xi) \right] \\
&= \prod_{s=1}^m \binom{n_s}{k} \sum_{\ell=0}^{n_1+\dots+n_m-mk} \binom{n_1+\dots+n_m-mk}{\ell} (-1)^\ell \frac{\tilde{G}_{\ell+mk+1, q^{-1}}}{\ell+mk+1}.
\end{aligned}$$

On the other hand, by symmetric properties of Bernstein polynomials and (2.15), we have

$$\begin{aligned}
I_4 &= \int_{\mathbb{Z}_p} \prod_{s=1}^m B_{n_s-k, n_s}(1-\xi) d\mu_{-q}(\xi) \\
&= \binom{n}{k} \sum_{\ell=0}^{mk} \binom{mk}{\ell} (-1)^{mk+\ell} \int_{\mathbb{Z}_p} (1-\xi)^{n_1+\dots+n_m+\ell} d\mu_{-q}(\xi) \\
&= \prod_{s=1}^m \binom{n_s}{k} \sum_{\ell=0}^{mk} \binom{mk}{\ell} (-1)^{mk+\ell} \left([2]_q + q^2 \frac{\tilde{G}_{n_1+\dots+n_m+\ell+1, q^{-1}}}{n_1+\dots+n_m+\ell+1} \right) \\
&= \begin{cases} [2]_q + q^2 \frac{\tilde{G}_{n_1+\dots+n_m+1, q^{-1}}}{n_1+\dots+n_m+1}, & k=0 \\ \prod_{s=1}^m \binom{n_s}{k} \sum_{\ell=0}^{mk} \binom{mk}{\ell} (-1)^{mk+\ell} \left([2]_q + q^2 \frac{\tilde{G}_{n_1+\dots+n_m+\ell+1, q^{-1}}}{n_1+\dots+n_m+\ell+1} \right), & k \neq 0. \end{cases}
\end{aligned}$$

Equating I_3 and I_4 results in an interesting identity for q -analogue of Genocchi polynomials with weight 0.

Theorem 8. *The identity*

$$\begin{aligned} \sum_{\ell=0}^{n_1+\dots+n_m-mk} \binom{n_1+\dots+n_m-mk}{\ell} (-1)^\ell \frac{\tilde{G}_{\ell+mk+1,q^{-1}}}{\ell+mk+1} \\ = \begin{cases} [2]_q + q^2 \frac{\tilde{G}_{n_1+\dots+n_m+1,q^{-1}}}{n_1+\dots+n_m+1}, & k=0 \\ \sum_{\ell=0}^{mk} \binom{mk}{\ell} (-1)^{mk+\ell} \left([2]_q + q^2 \frac{\tilde{G}_{n_1+\dots+n_m+\ell+1,q^{-1}}}{n_1+\dots+n_m+\ell+1} \right), & k \neq 0 \end{cases} \end{aligned}$$

is true.

3. OTHER IDENTITIES

In this section, we consider Kim's p -adic q -log gamma functions related to the q -analogue of Genocchi polynomials.

Definition 1 ([4, 6]). For $x \in \mathbb{C}_p \setminus \mathbb{Z}_p$,

$$(1+x) \log(1+x) = x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1)} x^{n+1}.$$

Kim's p -adic locally analytic function on $x \in \mathbb{C}_p \setminus \mathbb{Z}_p$ can be defined as follows.

Definition 2 ([4, 6]). For $x \in \mathbb{C}_p \setminus \mathbb{Z}_p$,

$$G_{p,q}(x) = \int_{\mathbb{Z}_p} [x+\xi]_q (\log[x+\xi]_q - 1) d\mu_{-q}(\xi).$$

If $q \rightarrow 1$, then

$$G_{p,1}(x) \triangleq G_p(x) = \int_{\mathbb{Z}_p} (x+\xi) [\log(x+\xi) - 1] d\mu_{-q}(\xi). \quad (3.1)$$

Replacing x by $\frac{x}{n}$ in (3.1) leads to

$$(x+\xi) [\log(x+\xi) - 1] = (x+\xi) \log x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1)} \frac{x^{n+1}}{x^n} - x. \quad (3.2)$$

From (3.1) and (3.2), we can establish an interesting formula (3.3) which is useful for studying in the theory of the p -adic analysis and the analytic number.

Theorem 9. For $x \in \mathbb{C}_p \setminus \mathbb{Z}_p$,

$$G_p(x) = \left(x + \frac{\tilde{G}_{2,q}}{2} \right) \log x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1)(n+2)} \frac{\tilde{G}_{n+2,q}}{x^n} - x. \quad (3.3)$$

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